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A new spectroscopic mode identification method

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Abstract

We present the Fourier parameter fit method, a new spectroscopic mode identification method for non-radially pulsating stars. It relies on a direct fit to the Fourier parameters zero point, amplitude and phase across the line profile. The quality of the obtained fits is quantified by means of a χ^2-goodness-of-fit test and the optimization is carried out with genetic algorithms permitting the search over a large parameter space. The displacement field adopted for the calculation of the theoretical Fourier parameters is valid in the limit of slow rotation and takes into account first order effects of the Coriolis force. We tested the method for low-degree modes ($\ell \leq 4$) in a range of different settings of pulsational and stellar parameters. These tests showed that in general the azimuthal order m can be determined without ambiguity for modes with $\ell \leq 2$, whereas the determination of ℓ is generally affected by an error of ± 1.

1. Introduction

Asteroseismology relies on a precise fit of measured and theoretical frequency values. Obtaining a unique fit is especially difficult for δ Scuti or β Cephei stars, which mainly exhibit non-radial low-degree, low-order pulsation modes whose frequency spacing is far below the asymptotic limit. Moreover, for most δ Scuti stars we only observe a fraction of frequencies predicted by linear pulsation theory. Therefore, a simple matching of the observed frequency pattern with a theoretical pattern has been unsuccessful.

In order to overcome this problem, a large number of mode identification (MI) methods, both, for photometry and spectroscopy, have been developed. Whereas photometric MI techniques are only sensitive to the spherical degree ℓ of a pulsation mode, spectroscopic MI methods, which make use of the temporal change of a line profile, permit the determination of the azimuthal order m as well as other parameters, such as the intrinsic displacement amplitude or the stellar inclination angle. By combining light and velocity variations, information about the efficiency of convection can be acquired (Daszynska-Daszkiewicz et al. 2002)

The previous MI methods have one thing in common: no confidence intervals for the obtained fits can be given. The moment method (Balona 1986, Aerts et al. 1992, Briquet & Aerts 2003) and the pixel-by-pixel method (Mantegazza 2000) both quantify the goodness of fit with discriminants that are not directly related to the observational uncertainties. Therefore, it is often not possible to select between different obtained solutions if their discriminants are too close.

2. Computation of the line profiles

We have implemented the calculation of the pulsationally distorted absorption line profiles in a C-program code. Our description of the Lagrangian displacement field ξ is limited to linear pulsations of a slowly rotating star and takes into account the effects of the Coriolis force to the first order. Since deviations from spherical symmetry due to centrifugal forces are ignored, our formalism provides only a useful description of the resulting velocity field for pulsation modes where $\Omega/\omega < 0.5$ (Aerts & Waelkens 1993, Schrijvers et al. 1997). This limitation excludes a realistic modeling of rapidly rotating stars and low frequency g-modes. For higher frequency p-modes such as observed in many δ Scuti stars the given criterion is fulfilled and good approximation provided. The code is also capable of computing a displacement field and resulting line profiles and light variations, respectively, for a symmetry axis that is not aligned with the rotation axis, as it is observed in roAp stars (Kurtz 1982).

The computation of absorption line profiles is carried out by a numerical integration over the visible disk calculated from orthogonal projection of the three-dimensional displaced surface. Therefore, deviation from spherical symmetry are taken into account. Furthermore, we assume that the intrinsic line profile is a Gaussian, which may undergo equivalent width changes due to temperature variations.

3. The Fourier Parameter Fit method

In our method, the Fourier Parameter Fit (FPF) method, we directly fit the observed Fourier parameters with theoretical values by applying a χ^2-test. The main differences to previous similar MI methods are the following:

- We utilize *all* available information of the Fourier parameters, zero point, amplitude and phase across the line.
- The calculation of χ^2 enables us to *quantify the significance* of our obtained fits.
- The optimization is carried out with *genetic algorithms*, which permit the detection of local minima in a large multi-parameter space in much shorter computational time than a grid.

Time series spectra of high resolution (R > 40000) and high signal-to-noise ratio (S/N> 200) are required in order to enable a successful application of the FPF method. For multi-periodic stars, lengthy campaigns covering several months of spectroscopy accompanied by simultaneous photometric measurements are the best approach (e.g., Aerts et al. 2004).

The theoretical Fourier parameters are computed from a least-squares fit of typically ten synthetic mono-mode line profiles evenly sampled over one pulsation cycle. As for the observational Fourier parameters, here we also fix the frequency during the least-squares fit of the wavelength bins and only derive the zero point Z_λ, the amplitude A_λ and the phase ϕ_λ as a function of wavelength.

The observational variance σ_λ^2 of the intensity of every pixel across the profile must be derived very carefully, since the value of χ^2 is very sensitive to this variance. σ_λ^2 is generally not constant along the profile, but increases towards its center due to the lower signal present there. It can be directly derived from the error matrix of the multi-period least-squares fit or the residuals after prewithening of all frequencies.

Our approach is the following: we first determine the pulsationally independent stellar parameters $v \sin i$, equivalent width W_0 and intrinsic line width σ by fitting the observed zero point profile Z_λ with static rotationally broadened line profiles. We assume Z_λ to be independent of the pulsation, which is true for a low ratio of the radial velocity amplitude to

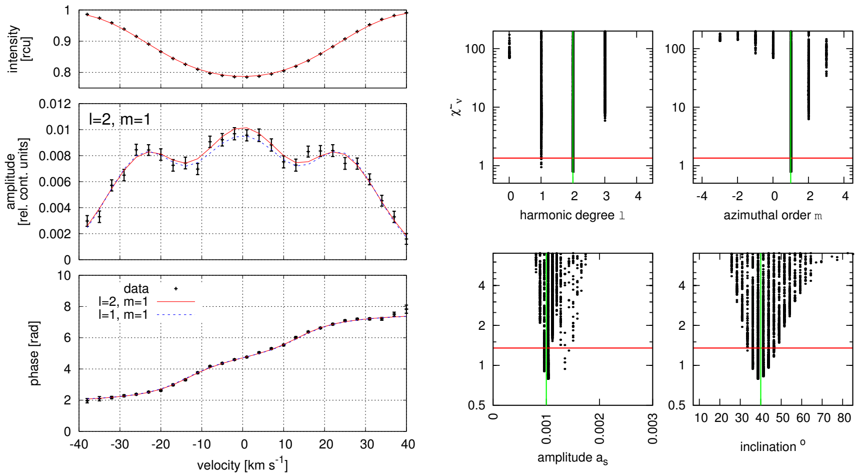

Figure 1: Application of the FPF method to synthetic line profile variations of a $\ell = 2, m = 1$ displacement field (300 spectra, S/N=200). The left panels show, from top to bottom, the zero point, amplitude and phase across the line of the input data (black crosses with error bars) and the two best fits (lines). The four panels at the right hand side show χ^2 with respect to ℓ (top left), m (top right), the pulsational amplitude (bottom left) and the inclination angle (bottom left). Every single dot represents a model computed during the genetic optimization. The vertical thinner line is at the position of the input parameter values and the horizontal red line denotes the 95 %-confidence limit.

$v \sin i$. During the optimization process we set the values of these parameters as variable in a narrow range set by their derived uncertainties.

The pulsational parameters of the modes are derived from the amplitude and phase across the line. The reduced χ^2_ν is calculated from complex amplitudes in order to combine amplitude and phase information in the following way

$$\chi^2_\nu = \frac{1}{2n_\lambda - m} \sum_{i=1}^{n_\lambda} \left[\frac{(A^o_{R,i} - A^t_{R,i})^2}{\sigma^2_{R,i}} + \frac{(A^o_{I,i} - A^t_{I,i})^2}{\sigma^2_{I,i}} \right], \quad (1)$$

where n_λ is the number of pixels across the profile, m is the number of free parameters, A^o and A^t denote observationally and theoretically determined values, respectively, $A_R = A_\lambda \cos\phi_\lambda$ and $A_I = A_\lambda \sin\phi_\lambda$ are the real and imaginary part of the complex amplitude, and σ^2 is the observational variance. Eq. (1) can be easily modified if the observational amplitude and phase from photometric passbands should be included for the calculation of χ^2_ν.

Since the amplitude and phase of a given wavelength bin are treated as independent variables, the variances are calculated from

$$\sigma^2_{R,\lambda} = \sigma(A_\lambda)^2 \cos^2\phi_\lambda + \sigma(\phi_\lambda)^2 A^2_\lambda \sin^2\phi_\lambda, \quad (2)$$

$$\sigma^2_{I,\lambda} = \sigma(A_\lambda)^2 \sin^2\phi_\lambda + \sigma(\phi_\lambda)^2 A^2_\lambda \cos^2\phi_\lambda. \quad (3)$$

In the case of a multi-periodic star, it is possible to optimize the sum of the χ^2_ν values of all present modes simultaneously taking common values of the pulsationally independent parameters inclination, $v \sin i$, σ and W. By doing so, especially the stellar inclination angle and henceforth also the intrinsic pulsation amplitudes of the modes can be better reproduced, since we take advantage of the fact that all observed modes are seen at the same aspect angle. Such an improvement for the MI in multi-periodic stars has also been acquired by Briquet & Aerts (2003) for the case of the moment method.

We carry out the search for the best fitting model by means of genetic optimization algorithms enabling the exploration of a large parameter space in much shorter computing time than can be done by grid calculations. The basis of a genetic code is a stochastically selected set of models, which evolves in time towards better models by favoring those having a higher fitness (lower χ^2_ν). The strength of genetic algorithms is especially their ability to explore many local minima simultaneously and consequently to locate the absolute minimum of a complex multi-parameter optimization problem.

Important diagnosis tools for determining the stability of the obtained solutions are diagrams, where for each computed model, χ^2 is plotted with respect to a tested parameter (see right-hand side of Fig. 1). Such diagrams help to estimate the uncertainty of the derived parameter values.

4. Tests for low-degree modes

We have tested the FPF method for low-degree pulsation modes ($\ell \leq 4$, $0 \leq m \leq \ell$ for each ℓ) for stellar model parameters resembling those of the δ Scuti star FG Vir (HD 106384, Breger et al. 2005, Daszynska-Daszkiewicz et al. 2005). These tests were carried out in a classical way by fitting simulated observations of line profile variations. We will here briefly summarize the obtained results.

The tests showed that we can generally derive the azimuthal order m unambiguously for input m-values of $0 \leq m \leq 2$. For higher m-values the uncertainty increases to ± 1. The value of ℓ is less constrained and generally affected by an uncertainty of ± 1. These numbers must be understood as crude estimates, since the precision to which ℓ and m can be determined strongly depends on the pulsational amplitude, the S/N and $v \sin i$. The visibility of low-degree modes in the line profile variations is favored for low projected rotational velocities, whereas high-degree modes are more easy seen at high values of $v \sin i$, i.e., the intensity variations in the profile have higher relative amplitude. In principle, the method can handle any value of ℓ, provided that the pixel-by-pixel amplitude is high enough.

The accuracy to which $v \sin i$ can be determined is in the range of 1 km s^{-1} for tested $v \sin i$-values between 10 and 100 km s^{-1}. Below $v \sin i$ =10 km s^{-1} the uncertainty in deriving $v \sin i$ rapidly increases due to the larger impact of the unknown intrinsic line width on the fit. The lower $v \sin i$-limit for the application of the FPF method is at about 5 km s^{-1}. An example fit to the Fourier parameters of synthetic profiles is shown the left-hand side of Fig. 1.

One of the results of our tests is that non-axisymmetric modes are better suited for deriving the inclination angle (see Fig. 2). For axisymmetric modes, the sensitivity of χ^2 with respect to the inclination is very small due to the fact that we cannot decouple the intrinsic amplitude and the inclination and, since we do not know the intrinsic amplitude, we can scale it such that any inclination can be fitted satisfactorily.

The best approach in order to derive a more precise inclination value is a simultaneous fit to the Fourier parameters of all detected non-axisymmetric pulsation modes with common values of $v \sin i$, intrinsic line width, equivalent width and inclination. Such a multi-mode fit can also constrain other pulsational parameters better, but due to the large number of free parameters it is more time consuming.

A comparison with the moment method and the pixel-by-pixel method showed that especially m can be determined with much better precision by the FPF method. This is mainly due to the fact that for the determination of the pulsational parameters, only amplitude and phase across the line profile are taken into account whereas the pulsationally independent zero point profile is ignored.

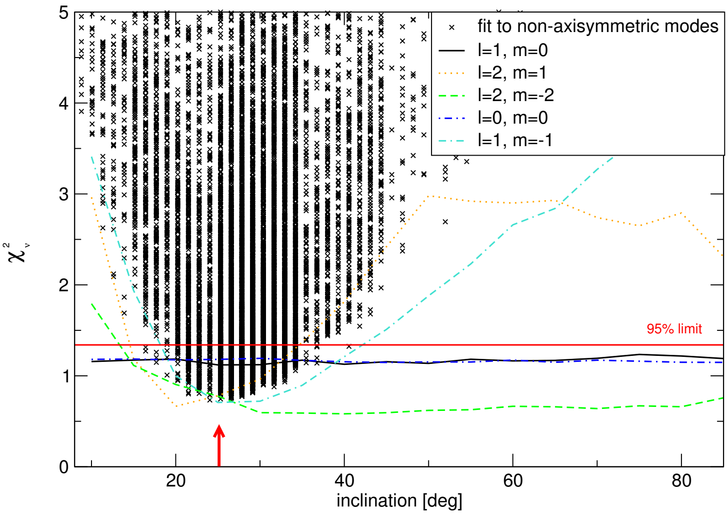

Figure 2: Determination of the stellar inclination angle from the application of the FPF method to synthetic line profiles ($v \sin i = 30$ km s^{-1}, $i = 25°$). The derived χ^2_ν-values are shown as a function of the inclination for different displacement fields (solid lines) and a multi-mode fit to the non-axisymmetric modes (crosses). Axisymmetric modes are not well suited for deriving the inclination angle. A simultaneous fit to all non-axisymmetric modes provides the best estimate of the inclination.

Our tests showed that the FPF method is ideally suited for the identification of pulsation modes of slowly to intermediate rotating stars which mainly exhibit low-degree modes. The high sensitivity with respect to the identification of m makes it an excellent tool complementary to the identification method by non-adiabatic observables (Daszynska-Daszkiewicz et al. 2002), where only ℓ can be determined.

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