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Supernovae Ia in the Early Universe

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Abstract

Measurements of the luminosity of supernovae Ia for high red shifts suggest a finite cosmological constant. It is shown that the luminosity variation can be understood within a Friedmann model of the universe, if the early supernovae start from white dwarf stars with a silicon core.

1. Introduction

If a binary stellar system consists of a giant star and a white dwarf (WD), gas can flow from the giant star to the WD. If the mass flow is not too slow, then soon the Chandrasekhar mass limit M_{ch} is reached: The WD collapses, carbon (^{12}C) and oxygen (^{16}O) are burned to ^{56}Ni, which is transformed to ^{56}Fe by two beta-decays. The consequence is a supernova explosion of type Ia (SN Ia), where the WD is destroyed totally. Thus the energy of the explosion

$$E = E_{tn} + 0.5E_g \qquad (1)$$

is given by the thermonuclear energy E_{tn} for the process (C,O) $\rightarrow$ Fe, diminished by the binding energy $-0.5E_g$. Here E_g means the gravitation energy.

Using a schematic model of SN Ia one starts with a WD of mass $M = 1.3M_s$, where M_s means the solar mass. Under the assumption that the gravitation pressure is in equilibrium with the pressure of a degenerate electron gas, the radius a and the binding energy

$$-0.5E_g = 2.501 \times 10^{-11}\,\mathrm{J}(\mathrm{M/kg})^2(\mathrm{a/m})^{-1} = 1.672 \times 10^{50}\,\mathrm{Jm\ a}^{-1} \tag{2}$$

can be determined.

The Chandrasekhar mass M_{ch} depends on the specific electron number N_e/M, where N_e is the total electron number and M is the mass of the star.

$$M_{ch} = 1.557 \times 10^{-53} M_s (N_e/M)^2 \tag{3}$$

For a mass abundance $X(^{12}\mathrm{C}) = X(^{16}\mathrm{O}) = 0.5$ this means

$$N_e/M = 3.012 \times 10^{26}\ \mathrm{kg}^{-1}, \quad M_{ch} = 1.412\ M_s. \tag{4}$$

As the WD expands during the carbon and the oxygen burning, the material in the neighbourhood of the surface is burned partially only or not at all. In a simplified version it shall be assumed that 75 per cent of the mass are burned finally to ^{56}Fe and 25 per cent do not change at all in their nuclear composition. In this case one gets the radius a and the energy E_0.

$$\begin{aligned} a &= 2846\ \mathrm{km}, 0.5E_g = -5.875 \times 10^{43}\,\mathrm{J}, \quad E_{tn} = 1.8985 \times 10^{44}\,\mathrm{J} \\ E_0 &= E_{tn} + 0.5E_g = 1.311 \times 10^{44}\,\mathrm{J}, \quad M_{B0} = -19.7 \end{aligned} \tag{5}$$

E_0 is the total energy of a SN Ia explosion. As the time dependence of the luminosity is very similar for various SNe, the maximum luminosity can be characterized by the same B-region magnitude M_{B0}.

The small variation of the mass M_{ch} allows to use the SNe Ia as standard candles to measure the distance of galaxies and to investigate the space-time geometry of the universe. Assuming a Friedmann-Robertson-Walker model of a matter dominated universe, the apparent magnitude m_B is given by the expression

$$\begin{aligned} m_B = M_B + 25 + 5l(z) - 5\log(H/\mathrm{km\ s}^{-1}\mathrm{Mpc}^{-1}) \\ + 5\log\{(2/z\Omega^2)[\Omega(z-1) + 2 + (\Omega - 2)(1 + z\Omega)^{1/2}]\}, \end{aligned} \tag{6}$$

where Ω is the density parameter, H is Hubble's constant, z is the red shift and l means the expression

$$l(z) = \log(zc/\mathrm{km\ s}^{-1}).$$

In a Hubble diagram m_B is plotted as a function of l. For a nearly flat universe

$$M_B = -19.7, \quad H = 55\,\mathrm{km\ s}^{-1}, \quad \Omega = 1.005 \pm 0.005$$

one gets a weakly curved line

$$m_B = 0.23l^2 + 2.99l + 1.048 \quad \text{for} \quad l < 4.94 \tag{7}$$

For red shifts $z < 0.29$ the SN Ia are concentrated to the function (7). The experiments of Perlmutter and coworkers, published in Nature [1] and later in other periodicals [2], show strong deviations from the function (7) for large red shifts ($0.3 < z < 0.9$). The mean behaviour can be approximated by the function

$$m_B = 6.99l - 13.1 \qquad \text{für } l > 4.94 \tag{8}$$

Under the assumption that also the early SNe are standard candles of the magnitude M_{B0}, one may deduce unusual values for the density parameter Ω and for a finite cosmological constant Λ. On the other hand, an interpretation which seems to be more natural, is the following: For higher red shifts too, the space-time geometry can be described by a Friedmann model, but the luminosity of the early SNe is smaller, because the precursor of a WD is a red giant star: In the central region helium is burned to carbon and oxygen, in the outer region hydrogen is burned to helium. This zone burning lasts for a longer time within old stars (population II), because on account of the small ^{12}C-abundance the contribution of the CNO-cycle to the hydrogen burning is reduced. Thus there is time enough that carbon and oxygen in the stellar core are partially transformed to silicon (^{28}Si). As the reaction chain Si $\rightarrow$ Fe has a smaller Q-value than the other processes (C $\rightarrow$ Fe, 0 $\rightarrow$ Fe), the old SNe will have a smaller luminosity than the younger SNe, which consist of carbon and oxygen only.

2. The Silicon Core

If the difference between the functions (8) and (9) is due to a decrease in the total SN energy, and if the time-dependence of the luminosity is independent of the energy, then one gets the relation

$$\log(E/E_0) = 0.092l^2 - 1.6l + 5.66. \tag{9}$$

The dependence of the ratio E/E_0 on the silicon content of the SN precursor can be evaluated by the investigation of a star with the mass abundances

$$X(^{12}\text{C}) = X(^{16}\text{O}) = 0.5\,Y, \quad X(^{28}\text{Si}) = 1 - Y. \tag{10}$$

The results are shown in Table 1. The WD radius, the Chandrasekhar mass limit and the specific electron number depend weakly on the variable Y.

Table 1. White dwarf radius a, mass limit M_{ch}, specific electron number N_e/M and total SN energy E for various silicon mass abundances $1-Y$

Y	(a/km)	(M_{ch}/M_s)	$((N_e/M)/10^{26}\mathrm{kg}^{-1})$	(E/E_0)
1	2846	1.4119	3.0115	1
0.8	2851	1.4123	3.0119	0.7451
0.6	2856	1.4127	3.0123	0.4901
0.4	2862	1.4130	3.0127	0.2349

Table 2. Relative SN energy E/E_0, silicon mass abundance $1-Y$, age $\bar{t}_0$ of the universe in the rest frame of the SN in their dependence on $l(z)$

l	(E/E_0)	$1-Y$	$(\bar{t}_0/\mathrm{Ga_{tr}})$	$((t-\bar{t}_0)/\mathrm{Ga_{tr}})$
4.94	1	0	7.78	4.06
5.0	0.9103	0.0704	7.38	4.46
5.1	0.7800	0.1726	6.56	5.28
5.2	0.6712	0.2579	5.74	6.10
5.3	0.5801	0.3295	4.86	6.98
5.4	0.5034	0.3896	3.99	7.85
5.5	0.4075	0.4647	3.20	8.64

Only the total SN energy varies significantly with the mass abundance of silicon.

The numbers in Table 1 can be interpolated by the relation

$$Y = 2.973 \times 10^{-6}\left(\frac{E}{E_0}\right)^3 + 8.404 \times 10^{-4}\left(\frac{E}{E_0}\right)^2 + 0.783\frac{E}{E_0} + 0.216. \tag{11}$$

Using the approximations (9) and (11) one finds the numerical values shown in Table 2. In the assumed Friedmann universe it is possible to determine the present age $t = 11.84\ \mathrm{Ga_{tr}}$ of the universe. $\mathrm{a_{tr}}$ is the tropical year 2000. $\bar{t}_0$ means the age of the universe in the rest frame of the light source at the time of the SN explosion. The last column in the table is an upper limit for the time difference between the emission and the observation of the light of the SN.

3. Conclusion

The strong variation of the SN luminosity with the silicon content of the SN precursor suggests that within the first two Gigayears of the cosmic evolution a WD contained 44 mass per cent silicon, and the hydrogen burning in the outer region of the corresponding red giant was dominated by the proton-proton cycles. For $\bar{t}_0$ between 2 and 8 gigayears the silicon abundance decreased in the stellar core because the shell of the red giant contained more and more carbon and nitrogen. The CNO-cycle contributes essentially; thus the zone burning becomes faster. For $\bar{t}_0 > 8$ Ga$_{\text{tr}}$ the WD does not contain silicon at all; the SN Ia becomes a standard candle with a total energy of $E_0 = 1.3 \times 10^{44}$ J. Tentatively this behaviour can be expressed by the relations

$$\begin{aligned}
&Y = 0.56 && \left(\frac{\bar{t}_0}{\text{Ga}_{\text{tr}}} < 2.1\right) \\
&Y = 0.0135\left(\frac{\bar{t}_0}{\text{Ga}_{\text{tr}}} - 2.1\right)^2 + 0.56 && \left(2.1 < \frac{\bar{t}_0}{\text{Ga}_{\text{tr}}} < 7.8\right) \\
&Y = 1 && \left(7.8 < \frac{\bar{t}_0}{\text{Ga}_{\text{tr}}} < 11.8\right)
\end{aligned} \tag{12}$$

Summarizing one cannot exclude density parameters $\Omega < 1$ or a finite cosmological constant. On the other hand it cannot be expected either that special values of these parameters could be derived from the apparent luminosity of supernova explosions.

References

[1] Perlmutter, S. et al.: Discovery of a supernova explosion at half the age of the universe. Nature **391**, 51 (1998).

[2] Perlmutter, S. et al.: Measurements of Omega and Lambda from 42 High-Redshift Supernovae. Preprint astroph/9812133.

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