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Alpha Particle Diffusion and Neutrino Production Within the Sun

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Abstract

The external properties of the sun represent strong boundary conditions for possible models of the solar interior. Even the variation of the particle diffusion is restricted by the observed intensity of high energy solar neutrinos

1. Introduction

The possibilities for alternative solar models are limited by the external properties of the sun: The radial distribution of the mass density and of the total pressure are determined by the assumption of static equilibrium, by the multi-pole oscillation frequencies, the radius and the mass of the sun. The chemical composition of the sun is characterized on the one hand by the initial mass abundances of hydrogen, helium and heavier elements. On the other hand the central region of the sun has a deficiency of 4.6×10^{55} protons and contains 1.16×10^{55} additional α-particles, which have been produced since the formation of the solar system. For a known distribution of hydrogen and helium the temperature and the radiation power L_{HH} of the various proton-proton fusion processes can be evaluated. The contribution of the CNO cycles is proportional to the mass abundance

$$X_6 = X(C) + X(N)$$

of carbon plus nitrogen. This number is determined by the total radiation power L of the sun

$$L_{\mathrm{HC}} = L - L_{\mathrm{HH}} \propto X_6, \quad L = 3.846(1) \times 10^{26}\,\mathrm{W},$$

where, here and later, the notation

$$3.846(1) = 3.846 \pm 0.001$$

is used.

In this way many parameters of a solar model are fixed already. Only the radial distribution for a given number of α-particles can be varied. Temperature and energy spectrum of the solar neutrinos depend sensitively on a change of the α-particle distribution by diffusion. In this connection the neutrinos are important because from the solar radiation power one can deduce a mean neutrino flux of $6.54 \times 10^{14}\mathrm{m}^{-2}\,\mathrm{s}^{-1}$. Experimentally roughly one half of this amount can be detected. In the case of the chlorine and gallium experiments the reduction can be interpreted as a neutrino oscillation. This means that a part of the electron neutrinos produced in the sun are transformed to muon and tauon neutrinos, which cannot induce an inverse beta decay in Cl- and Ga-nuclei on the earth. But in the Kamiokande experiment based on the electron-neutrino scattering there is no strong dependence on the special type of neutrino; in this experiment there is a preference to the high energy neutrinos from the ^{8}B decay. The boron production is sensitive to the central temperature and thus sensitive to the helium distribution in the central region of the sun. In this case a reduced neutrino flux indicates a central temperature smaller than the corresponding value in the standard solar model without diffusion. In a previous contribution [1] temperature and diffusion have been estimated. In this paper the helium distribution function shall be discussed in a more systematic way. All contributions to the total pressure and all HH fussion processes shall be taken into account. Finally the possible variations of a solar model receive a further restriction. Section 3 shows the consequences for the diffusion of helium in the radial region, where the thermonuclear reactions are concentrated. In Section 4 the particle distributions are varied and the results are summarized. A comparison of the expected neutrino fluxes with the experimental results favours a special distribution of the α-particles in the sun.

2. The Radial Distribution of Helium

The mass abundances X, Y and $1-X-Y$ of hydrogen (^{1}H), helium (^{4}He) and the other nuclides (^{2}H, ^{3}He, atomic number $Z > 2$) respectively, start with the initial values

$$X_0 = 0.727, \quad Y_0 = 0.256, \quad 1 - X_0 - Y_0 = 0.017. \tag{1}$$

The thermonuclear reactions in the sun led to the present mass abundances

$$X = X_0 - \hat{Y}\,, \quad Y = Y_0 + \hat{Y}\,, \quad 1 - X - Y = 0.017. \qquad (2)$$

They are characterized by the mass abundance $\hat{Y}$ of ^{4}He, which has been produced during the history of the sun. The distribution of the heavier elements does not change by the HH and HC processes. The present particle number densities n_p of protons and n_α of α-particles can be expressed by the mass density ρ

$$n_p = 5.97538 \times 10^{26}\,\mathrm{kg}^{-1}\rho X, \quad n_\alpha = 1.50456 \times 10^{26}\,\mathrm{kg}^{-1}\rho Y \qquad (3)$$

Since 4.64 $\mathrm{Ga_{tr}}$ the sun is a star on the main sequence of the Hertzsprung-Russel diagram. As a time unit the tropical year 2000 is used

$$1\mathrm{a_{tr}} = 3.15569254 \times 10^7\mathrm{s}, \quad 1\mathrm{Ga_{tr}} = 10^9\mathrm{a_{tr}}.$$

It shall be assumed that the electromagnetic radiation power L_t increased from 72 to 100 per cent of its present value

$$L_t = 3.846 \times 10^{26}\ \mathrm{W}\left(0.72 + 0.28\frac{t}{4.64\mathrm{Ga_{tr}}}\right). \qquad (4)$$

Then one gets a total number

$$\begin{aligned}\hat{N}_\alpha &= \int \hat{n}_\alpha dV \\ &= 1.50456 \times 10^{26}\,\mathrm{kg}^{-1}\int \rho\hat{Y}\,dV = 1.156 \times 10^{55} \qquad (5)\end{aligned}$$

of additional α-particles, if the early production of thermonuclear energy is dominated by HH processes and HC processes can be neglected. It can be shown numerically that the specific radiation power is of the type

$$\begin{aligned}&\frac{dL_r}{dM} = \frac{1}{\rho}\frac{dL_r}{dV} = 1.82(7) \times 10^{-3}\frac{\mathrm{W}}{\mathrm{kg}}\ \exp\left(-40x^{5/3}\right), \\ &x = r/R_s, \quad R_s = 6.9626 \times 10^8\,\mathrm{m}, \qquad (6)\end{aligned}$$

where r is the distance from the solar center. L_r means the radiation power at the distance r from the center. R_s is the solar radius. The thermonuclear reactions are restricted to the central region of the sun ($x < 0.36$).

It can be assumed that the mass abundance of the additional α-particles can be expressed by the function (6)

$$\begin{aligned}\hat{Y} &= Y_1(\beta)\ \exp\left(-\beta x^{5/3}\right) \quad &&\text{for } x \le 0.36 \\ \hat{Y} &= 0, &&\text{for } x > 0.36\end{aligned} \qquad (7)$$

where the constant Y_1 is fixed by the integral (5)

$$Y_1(\beta) \int_0^{0.36} \rho(x)\left[\exp\left(-\beta x^{5/3}\right)\right] x^2 dx = 18.108 \text{ kgm}^{-3}. \quad (8)$$

In a solar model without diffusion ($\beta = 40$), the central temperature has its maximum value. If diffusion is possible ($\beta < 40$), then the central temperature becomes smaller.

The choice of the mass density ρ is not arbitrary, but it is restricted by the fact that in an equilibrium state ρ and the gravitation pressure P are correlated with each other and with the velocity v_s of sound

$$\begin{aligned} r^2 dP &= -G_N M_r \rho dr, & M_r &= 4\pi \int_0^r \rho(s/R_s) s^2 ds \\ v_s &= (5P/3\rho)^{1/2}, & G_N &= 6.67259 \times 10^{-11} \text{ kg}^{-1}\text{m}^3\text{s}^{-2}, \quad (9) \end{aligned}$$

where G_N is Newton's constant. Eq. (9) for v_s holds in a totally ionized material. This velocity can be evaluated from the frequencies of solar multipole vibrations. A mass density distribution, for which these values of v_s can be reproduced, has been proposed by Stix [2]. For the central region of the sun ($x \leq 0.36$) the values of ρ, M_r/M_s, P and v_s are given in Table 1, where $M_s = 1.9889 \times 10^{30}$ kg means the solar mass. The notation

$$aEn = a \times 10^n$$

Table 1. Mass density ρ, mass M_r, total pressure P and velocity v_s of sound in the central region of the sun

(r/R_s)	$(\rho/\text{kg m}^{-3})$	(M_r/M_s)	(P/P_a)	(v_s/kms^{-1})
0.00	1.530E5	0	2.372E16	508.3
0.02	1.481E5	8.531E-4	2.310E16	509.8
0.04	1.359E5	6.472E-3	2.142E16	512.5
0.06	1.198E5	2.019E-2	1.905E16	514.9
0.08	1.029E5	4.344E-2	1.642E16	515.6
0.10	8.769E4	7.626E-2	1.380E16	512.1
0.12	7.403E4	0.1178	1.136E16	505.8
0.14	6.220E4	0.1667	9.197E15	496.4
0.16	5.195E4	0.2212	7.334E15	485.1
0.18	4.309E4	0.2795	5.751E15	471.7
0.20	3.547E4	0.3397	4.455E15	457.5
0.22	2.902E4	0.4000	3.415E15	442.8
0.24	2.355E4	0.4590	2.602E15	426.1
0.26	1.905E4	0.5155	1.970E15	415.2
0.28	1.533E4	0.5686	1.484E15	401.6
0.30	1.225E4	0.6178	1.114E15	389.3
0.32	9.750E3	0.6626	8.349E14	377.8
0.34	7.756E3	0.7030	6.262E14	366.8
0.36	6.176E3	0.7391	4.702E14	356.2

Table 2. The mass abundances X and Y of ^{1}H and ^{4}He, the partial pressures P_γ, P_b and P_e of radiation, ions and electrons, respectively, for $\beta = 36$

(r/R_s)	X	Y	(P_γ/P_a)	(P_b/P_a)	(P_e/P_a)
0.00	0.399	0.584	1.185E13	1.016E16	1.355E16
0.02	0.416	0.567	1.136E13	9.957E15	1.313E16
0.04	0.450	0.533	1.042E13	9.347E15	1.206E16
0.06	0.491	0.492	9.304E12	8.436E15	1.061E16
0.08	0.535	0.448	8.102E12	7.370E15	9.037E15
0.10	0.576	0.407	6.683E12	6.269E15	7.520E15
0.12	0.612	0.371	5.385E12	5.215E15	6.141E15
0.14	0.643	0.340	4.219E12	4.255E15	4.937E15
0.16	0.667	0.316	3.261E12	3.415E15	3.915E15
0.18	0.685	0.298	2.469E12	2.691E15	3.058E15
0.20	0.699	0.284	1.862E12	2.092E15	2.360E15
0.22	0.709	0.274	1.399E12	1.609E15	1.805E15
0.24	0.715	0.268	1.070E12	1.229E15	1.372E15
0.26	0.720	0.263	8.141E11	9.320E14	1.037E15
0.28	0.723	0.260	6.362E11	7.030E14	7.799E14
0.30	0.724	0.259	4.835E11	5.283E14	5.848E14
0.32	0.726	0.257	3.802E11	3.965E14	4.380E14
0.34	0.726	0.257	3.007E11	2.977E14	3.282E14
0.36	0.727	0.256	2.382E11	2.237E14	2.463E14

is used. For a given value of β the mass abundances X and Y, the partial pressures P_γ, P_b and P_e of radiation, of baryons (nuclei, ions) and electrons, respectively, are determined for each distance r in the radial region under consideration. Table 2 shows the corresponding quantities for the parameter $\beta = 36$. In this region ($x = 0 \ldots 0.36$) by Pauli's principle the pressure P_e of the electrons increases by 3.9 ... 0.9 per cent compared to a classical gas of electrons.

Table 3 contains the temperature T (in units of Megakelvin), the relative atomic mass μ and the ratio L_r/L for $\beta = 36$. The temperature is fixed by the equilibrium condition that the sum

$$P_\gamma + P_b + P_e = P \qquad (10)$$

becomes equal to the gravitation pressure P. For the evaluation of the relative atomic mass μ it has been assumed that the gas is ionized totally. For $\beta = 36$ a value $X_6 = 0.0049$ is needed to reproduce the (electromagnetic) radiation power L. If the neutrino emission is taken into account, then the total radiation power

$$L_{\text{tot}} = 3.940 \times 10^{26} \text{ W} \qquad (11)$$

becomes by 2.45 per cent larger than the radiation power L.

Table 3. Temperature T, relative atomic mass μ, electromagnetic radiation power L_r at the distance r from the center for $\beta = 36$. Opacity $\kappa(\beta)$ for $\beta = 30$, 36 and 40 in units of $m^2\ kg^{-1}$

(r/R_s)	(T/MK)	μ	(L_r/L)	κ (30)	κ (36)	κ (40)
0.00	14.721	0.8074	0	–	–	–
0.02	14.568	0.7940	0.0066	–	–	–
0.04	14.258	0.7684	0.0531	0.036	0.078	0.116
0.06	13.859	0.7392	0.1506	0.045	0.080	0.108
0.08	13.388	0.7110	0.2889	0.069	0.098	0.120
0.10	12.759	0.6862	0.4525	0.095	0.117	0.132
0.12	12.088	0.6657	0.5961	0.123	0.137	0.146
0.14	11.373	0.6495	0.7239	0.149	0.157	0.160
0.16	10.664	0.6371	0.8234	0.176	0.178	0.178
0.18	9.947	0.6279	0.8895	0.202	0.200	0.197
0.20	9.270	0.6214	0.9378	0.222	0.217	0.213
0.22	8.630	0.6168	0.9632	0.236	0.230	0.226
0.24	8.070	0.6137	0.9811	0.253	0.246	0.242
0.26	7.538	0.6117	0.9893	0.266	0.260	0.256
0.28	7.087	0.6103	0.9950	0.295	0.290	0.287
0.30	6.617	0.6095	0.9973	0.319	0.314	0.312
0.32	6.231	0.6090	0.9989	0.328	0.324	0.322
0.34	5.876	0.6087	0.9998	0.361	0.358	0.356
0.36	5.544	0.6085	1	–	–	–

If the energy transport is due to radiation in thermal equilibrium, then the opacity $\kappa = \kappa(\beta)$ is given by the equation

$$L_r = -\frac{16\pi}{3\kappa\rho}\sigma_{\mathrm{SB}} r^2 \frac{\partial}{\partial r} T^4, \tag{12}$$

where σ_{SB} means the Stefan-Boltzmann constant. Table 3 shows that the opacity does not depend on the parameter β for $r > 0.16\ R_s$. For smaller distances r from the center $\kappa(36)$ is not essentially smaller than $\kappa(40)$. Therefore, the radiation equilibrium does not appear as a special problem in the region $\beta = 36 \ldots 40$.

3. Diffusion of α-Particles

The number $\hat{N}^{(0)}_{\alpha r}$ of additional α-particles, which have been produced in a spherical volume of radius r, is given by the expression

$$\hat{N}^{(0)}_{\alpha r} = 1.156 \times 10^{55} L_r / L. \tag{13}$$

The present number of additional α-particles in the same volume is equal to the integral

$$\hat{N}_{\alpha r} = 1.50456 \times 10^{26}\,\mathrm{kg}^{-1} \times 4\pi \int_0^r \hat{Y}(s/R_s)\rho s^2 ds. \tag{14}$$

Both numbers are shown in table 4 for $\beta = 36$. Here the considerations are restricted to the central region of the sun because 98 % of the radiation power and of the α-particle production arise from $r < 0.24\,R_s$. The difference between the numbers (13) and (14) can be expressed by the particle flux $j_\alpha(r/R_s, t)$ of α-particles

$$\hat{N}^{(0)}_{\alpha r} - \hat{N}_{\alpha r} = 4\pi r^2 \int_0^{t_1} j_\alpha(r/R_s, t)dt, \quad t_1 = 4.64\mathrm{Ga_{tr}}$$

$$j_\alpha(r/R_s, t) = (0.72 + 0.28t/t_1)\, j_{\alpha 1}(r/R_s), \tag{15}$$

where $j_{\alpha 1}(r/R_s)$ is the present flux of α-particles in radial direction

$$\hat{N}^{(0)}_{\alpha r} - \hat{N}_{\alpha r} = 3.44\pi t_1 r^2 j_{\alpha 1}(r/R_s). \tag{16}$$

Table 4 shows these values of the α-particle flux for $\beta = 36$. They may be compared with theoretical estimates of the diffusion current density j_d evaluated from the change $\Delta\lambda$ of the mean free path λ in radial direction

$$\sigma_p = \frac{1}{4}\sigma_\alpha = 2.18\left(2\alpha_f \frac{\hbar c}{M_\alpha v_\alpha^2}\right)^2$$

$$\lambda = \frac{1}{n_p\sigma_p + n_\alpha\sigma_\alpha}, \quad v_\alpha = \left(\frac{3}{M_\alpha} k_B T\right)^{1/2}. \tag{17}$$

Here σ_p and σ_α are the scattering cross sections of αp and $\alpha\alpha$ scattering orthogonal to the radial direction, respectively, if the target nuclei are screened by a gas of free electrons. α_f is the fine-structure constant. M_α means the mass and v_α the mean velocity of the α-particles. n_e, n_p and n_α

Table 4. Numbers $\hat{N}^{(0)}_{\alpha 0}$ and $\hat{N}_{\alpha r}$ of α-particles produced and now present, respectively, within a sphere of radius r. Present current density $j_{\alpha 1}$ and estimated diffusion flux j_d of α-particles for $\beta = 36$

(r/R_s)	$\hat{N}^{(0)}_{\alpha r}$	$\hat{N}_{\alpha r}$	$(j_{\alpha 1}/\mathrm{m^{-2}s^{-1}})$	$(j_d/\mathrm{m^{-2}s^{-1}})$
0.04	6.139E53	5.713E53	3.475E19	2.426E19
0.08	3.338E54	3.100E54	4.838E19	2.608E19
0.12	6.888E54	6.420E54	4.237E19	1.845E19
0.16	9.515E54	9.034E54	2.449E19	1.228E19
0.20	1.084E55	1.053E55	9.902E18	8.393E18
0.24	1.134E55	1.121E55	2.918E18	5.965E18

are the particle number densities of electrons, protons and α-particles, respectively. Contributions of the order $(\hbar n_e^{1/3}/M_\alpha v_\alpha)^2$ have been neglected in the expression (17) for the cross sections. For the diffusion current density one finds

$$j_\alpha = \frac{1}{6} n_\alpha v_\alpha, \quad j_d = 2j_\alpha = \frac{1}{3\lambda} n_\alpha v_\alpha \lambda$$
$$j_d = \frac{1}{6.54\pi} \left(\frac{M_\alpha}{2\alpha_f \hbar c} \right)^2 \frac{n_\alpha v_\alpha^5}{(n_p + 4n_\alpha)^2} \left(-\frac{\partial n_p}{\partial r} - 4\frac{\partial n_\alpha}{\partial r} \right). \tag{18}$$

For $\beta = 36$ both current densities, $j_{\alpha 1}$ and j_d, are in the same order of magnitude. Thus the numbers in Table 4 suggest a solar model with $\beta = 36$.

4. Conclusion

Possible variations of the solar model are restricted strongly by the external properties of the sun (mass, radiation power, multipole vibration frequencies, total number $\hat{N}_\alpha$ of α-particles produced within the sun). Therefore, only the parameter β, which characterizes the present distribution of the α-particles, can be changed. The consequences of such a variation are shown in Table 5. For $\beta = 30 \ldots 36$ the present current density $j_{\alpha 1}$ is in the same order of magnitude as the theoretically estimated diffusion flux j_d. The value $\beta = 40$ however, can be excluded, because in this case the current density $j_{\alpha 1}$ is too small; it even becomes negative for $r = 0.08\,R_s$. The radiation power L_{HH} arising from various *pp* cycles remains nearly unchanged as a function of the parameter β. Thus the mass abundance X_6 in the central region of the sun is fixed by the experimental radiation power L. An effective vector coupling constant G_V' for the pro-

Table 5. Ratio of the α-particle current densities, the contribution L_{HH} of the pp-processes to the radiation power, the mass abundance X_6 of carbon and nitrogen, the central temperature T_c, the total neutrino flux ϕ_ν, flux ϕ (B) of neutrinos from the ^{8}B-decay, the neutrino reaction rates $\phi\sigma$ for the target nuclei ^{37}Cl and ^{71}Ga. Dependence on the parameter β

β	$(j_{\alpha 1}/j^d)$	$(L_{\mathrm{HH}}/\mathrm{W})$	X_6	(T_c/MK)
30	0.5 ... 2.3	3.634E26	82E-4	13.848
36	0.5 ... 2.3	3.628E26	49E-4	14.721
40	−0.1 ... 0.6	3.617E26	34E-4	15.411
β	$(\phi_\nu/\mathrm{m}^{-2}\mathrm{s}^{-1})$	$(\phi(B)/\mathrm{m}^{-2}\mathrm{s}^{-1})$	$(\phi\sigma(\mathrm{Cl})/\mathrm{snu})$	$(\phi\sigma(\mathrm{Ga})/\mathrm{snu})$
30	6.5426E14	0.8E10	3.6	135
36	6.5431E14	2.2E10	5.1	139
40	6.5439E14	4.5E10	7.7	148
exp.	–	2.7(5)E10	2.6(4)	70(8)

cess $p(p, de^+\nu)$ must be smaller than $1.03 G_V$, otherwise one gets $L_{HH} > L$ even for $X_6 = 0$. For $G'_V = G_V$ and $\beta = 36 \ldots 40$ the corresponding values of X_6 cannot be excluded. For $\beta = 30$ the abundance X_6 becomes too high. Thus only $\beta = 36$ can be accepted.

This means that the central temperature becomes $T_c = 14.7$ MK rather than 15.4 MK ($\beta = 40$, standard solar model without diffusion). The total radiation power

$$L_{\text{tot}} = 3.94 \times 10^{26}\ \text{W}$$

includes the neutrino radiation too. L_{tot} and the total flux

$$\phi_\nu = 6.54 \times 10^{14} \nu/\text{m}^2\text{s}$$

of solar neutrinos, both are nearly independent of the parameter β. The special neutrino flux $\phi(B)$ arising from boron decay has been evaluated under the assumption

$$\phi(B) \propto T_c^{16}.$$

For $\beta = 36$ this flux is consistent with the experimental values (Kamiokande collaboration [3]). For $\beta = 30$ the neutrino flux is too small; for $\beta = 40$ it would be too large. As the Kamiokande experiment is based on the elastic νe scattering, which should be roughly independent of the special neutrino generation, a parameter $\beta = 36$ is favoured.

The reaction rates $\phi\sigma$ for the chlorine and the gallium experiment are given in solar nutrino units (snu $= 10^{-36}\text{s}^{-1}$ per target nucleus). For $\beta = 36$ both reaction rates $\phi\sigma$(Cl) and $\phi\sigma$(Ga) reach one half of the experimental values (references [4] and [5]). The reduction of the reaction rates is an indication for finite rest masses of the neutrinos: Either the neutrinos have a finite magnetic moment, which reduces the energy and the reaction rates of them; or by neutrino oscillations the electron neutrinos are partially transformed to muon and tauon neutrinos, which cannot induce inverse beta decays of Cl and Ga isotopes. The Kamiokande experiment however, can be reproduced quantitatively by a solar model with a diffusion parameter $\beta = 36$.

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